stress strength estimation for a new modified Weibull distribution and its applications

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Abstract

This paper investigates the estimation and monitoring process of stress strength parameters for a new modified Weibull distribution. The mentioned density was provided in mathematical form and relating figures for its main characteristics including probability density function and cumulative distribution function. Thereafter, the parameter is calculated. The likelihood function is maximized and accordingly, not only maximum likelihood estimations but the corresponding Fisher information matrix are also provided in detail. The monitoring process is described and hereafter, based on some numerical results, the performances of presented concepts are also examined.

Introduction

The stress-strength parameter which is defined as $R = P(X < Y)$ is a measure of system failure based on stress X exceeding a strength Y and proposed by Birnbaum's article in 1956. In fact when stress X exceeds the strength Y, system will be out of control. Nowadays Estimating and controlling this parameter has received considerable attention in the statistical literature.

SZMW distribution

Sarhan and Zaindin introduced a three-parameter distribution which known as Modified Weibull (SZMW) distribution. Its PDF and CDF are respectively

$$f(x) = (\alpha + \beta\gamma x^{\gamma-1}) \exp\{-\alpha x - \beta x^{\gamma}\}, \quad x > 0 \tag{1}$$

and

$$F(x) = 1 - \exp\{-\alpha x - \beta x^{\gamma}\}, \quad x > 0, \tag{2}$$

where $\alpha, \beta, \gamma > 0$. This distribution generalizes the following distributions: exponential, Rayleigh, linear failure rate, and Weibull.

Stress-Strength Parameter

Now suppose that $X \sim SZMW(\alpha_1, \beta_1, \gamma_1)$ and $Y \sim SZMW(\alpha_2, \beta_2, \gamma_2)$ be independent random variable which follows SZMW distribution. The stress-strength parameter R is define as

$$
\begin{aligned}
R = P(X < Y) &= \int_0^\infty (\alpha_1 + \beta\gamma x^{\gamma-1}) \exp\{-(\alpha_1+\alpha_2)x - 2\beta x^\gamma\} dx \\
&= \int_0^\infty (-\alpha_2 - \beta\gamma x^{\gamma-1} + \alpha_1 + \alpha_2 + 2\beta\gamma x^{\gamma-1}) \\
&\times \exp\{-(\alpha_1+\alpha_2)x - 2\beta x^\gamma\} dx
\end{aligned}
$$

$$= -\alpha_2 \int_0^\infty \exp\{-(\alpha_1+\alpha_2)x - 2\beta x^\gamma\}dx$$
$$- \beta\gamma \int_0^\infty x^{\gamma-1} \exp\{-(\alpha_1+\alpha_2)x - 2\beta x^\gamma\}dx$$
$$+ \int_0^\infty (\alpha_1+\alpha_2+2\beta\gamma x^{\gamma-1}) \exp\{-(\alpha_1+\alpha_2)x - 2\beta x^\gamma\}dx.$$

Now if We define

$$m_k^{\alpha,\beta} = \int_0^\infty x^k S_{\alpha,\beta}(x)dx,$$

the equation above will simplifies to

$$R = P(X<Y) = 1 - \alpha_2 m_0^{\alpha_1+\alpha_2,2\beta} - \beta\gamma m_{\gamma-1}^{\alpha_1+\alpha_2,2\beta}. \quad (3)$$

Maximum Likelihood Estimation of R

Let $X = (X_1, \cdots, X_n)$ and $Y = (Y_1, \cdots, Y_n)$ be the samples from SZMW distribution with different shape parameter. The log-likelihood function can be written as follows,

$$
\begin{aligned}
l(\alpha_1, \alpha_2, \beta, \gamma, x, y) &= \ln L(\alpha_1, \alpha_2, \beta, \gamma, x, y) \\
&= \sum_{i=1}^{n} \ln\{\alpha_1\alpha_2 + \beta\gamma(\alpha_1 y_i^{\gamma-1} + \alpha_2 x_i^{\gamma-1}) + (\beta\gamma)^2 (x_i y_i)^{\gamma-1} \\
&- \alpha_1 \sum_{i=1}^{n} x_i - \alpha_2 \sum_{i=1}^{n} y_i - \beta\Big(\sum_{i=1}^{n} x_i^{\gamma} + \sum_{i=1}^{n} y_i^{\gamma}\Big). \qquad (4)
\end{aligned}
$$

Maximum Likelihood Estimation of R

Which the estimation of the α_1, α_2, β and γ are obtained by equating the derivative of the above expression with zero toward each of these parameters.
by using invariance property of MLE, the stress-strength parameter defined in 4 can be rewritten as follows,

$$\hat{R} = P(X < Y) = 1 - \hat{\alpha_2} m_0^{\hat{\alpha_1}+\hat{\alpha_2},2\hat{\beta}} - \hat{\beta}\hat{\gamma} m_{\hat{\gamma}-1}^{\hat{\alpha_1}+\hat{\alpha_2},2\hat{\beta}}. \tag{5}$$

If the standard regularity conditions are established, the MLE estimator has an asymptotic normality distribution.

Simulation

we examine our methods of estimating the parameters $\alpha_1, \alpha_2, \beta, \gamma$.

Table: The performances of MLEs

θ	BIAS	MAE	MSE
$\alpha_1 = 1$	0.0023	0.0027	0.0019
$\alpha_2 = 2$	0.0017	0.0021	0.0022
$\beta = 1$	0.0016	0.0021	0.0024
$\gamma = 1$	0.0009	0.0006	0.0007

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