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INFERENCE ON MULTICOMPONENT STRESS-STRENGTH PARAMETER IN LOMAX DISTRIBUTION

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Abstract. Different estimation of multicomponent stress-strength parameter for Lomax distribution is considered, in veiw of frequentist and Bayesian inference. We derive the maximum likelihood estimation (MLE) and asymptotic confidence interval of multicomponent stress-strength parameter. Also, due to the lack of explicit form, the Bayes estimation of this parameter is obtained using two approximation method: Lindley's approximation and MCMC method. We compare different estimation methods using a Monte Carlo simulation.

1. Introduction

Statistical inference of the stress-strength parameter $R = P(Y < X)$ is a general problem of interest in reliability theory. The random variables Y and X are related to stress and strength, respectively. If at any time the applied stress is greater than its strength, the system fails. A multicomponent system is a system having more than one component. This system is composed of a common stress and k independent and identical strengths component. When s $(1 \leq s \leq k)$ or more of the components simultaneously survive, the system functions. [1] developed the multicomponent reliability as

$$\begin{aligned} R_{s,k} &= P[\text{at least } s \text{ of } (X_1, \dots, X_k) \text{ exceed } Y] \\ &= \sum_{p=s}^{k} \binom{k}{p} \int_{-\infty}^{\infty} [1 - F_X(x)]^p [F_X(x)]^{k-p} dF_Y(y), \end{aligned}$$

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when the common random stress Y with cdf $F_Y(.)$ subjected to $(X_1,\dots,X_k)$ which are independent and identically distributed random variables with cdf $F_X(.)$. Some authors have considered this problem. See for example [3, 4].

Lomax (Lo) distribution with the parameters α and λ, has the probability density function as $f(x)=\alpha\lambda(1+\lambda x)^{-(\alpha+1)},\ x,\alpha,\lambda>0$. In this paper, we obtain the different point and interval estimation of the $R_{s,k}$, when the stress and strengths are independent random variables from the Lomax distributions.

2. MLE of $R_{s,k}$

Suppose that $X\sim Lo(\alpha,\lambda)$ and $Y\sim Lo(\beta,\lambda)$ and they are independent random variables with unknown parameters α and β and common parameter λ. The multicomponent stress-strength reliability is given by

$$R_{s,k}=\sum_{p=s}^{k}\sum_{q=0}^{k-p}\binom{k}{p}\binom{k-p}{q}(-1)^q\frac{\beta}{\beta+\alpha(p+q)}.$$

In this case, we need to compute the MLE of the vector of parameters $\theta=(\alpha,\beta,\lambda)$ to compute MLE for $R_{s,k}$. Then, the likelihood function can be written as:

$$\begin{aligned}
L(\alpha,\beta,\lambda)&=\prod_{i=1}^{n}\Big(\prod_{j=1}^{k}f(x_{ij})\Big)g(y_i)\\
&=\prod_{i=1}^{n}\Big(\prod_{j=1}^{k}\alpha\lambda(1+\lambda x_{ij})^{-(\alpha+1)}\Big)\beta\lambda(1+\lambda y_i)^{-(\beta+1)}\\
&=\alpha^{nk}\lambda^{n(k+1)}\beta^{n}\Big(\prod_{i=1}^{n}\prod_{j=1}^{k}(1+\lambda x_{ij})^{-(\alpha+1)}\Big)\Big(\prod_{i=1}^{n}(1+\lambda y_i)^{-(\beta+1)}\Big)\\
&=\alpha^{nk}\beta^{n}\lambda^{n(k+1)}\Big(\prod_{i=1}^{n}\prod_{j=1}^{k}(1+\lambda x_{ij})^{-(\alpha+1)}\Big)\Big(\prod_{i=1}^{n}(1+\lambda y_i)^{-(\beta+1)}\Big),
\end{aligned}$$

where $\{X_{i1},\dots,X_{ik}\},\ i=1,\dots,n$, is a sample from $Lo(\alpha,\lambda)$ and $\{Y_1,\dots,Y_n\}$ is a sample from $Lo(\beta,\lambda)$. So the log-likelihood function can be derived by:

$$\begin{aligned}
\ell(\alpha,\beta,\lambda)&==nk\log\alpha+n\log\beta+n(k+1)\log\lambda-(\alpha+1)\sum_{i=1}^{n}\sum_{j=1}^{k}\log(1+\lambda x_{ij})\\
&-(\beta+1)\sum_{i=1}^{n}\log(1+\lambda y_i).
\end{aligned}$$

The MLE of α, β, which presented by $\widehat{\alpha}$, $\widehat{\beta}$ respectively, can be obtained as a solution of the following equation:

$$\frac{\partial\ell}{\partial\alpha}=\frac{nk}{\alpha}-\sum_{i=1}^{n}\sum_{j=1}^{k}\log(1+\lambda x_{ij})=0, \tag{2.1}$$

$$\frac{\partial\ell}{\partial\beta}=\frac{n}{\beta}-\sum_{i=1}^{n}\log(1+\lambda y_i)=0, \tag{2.2}$$

From (2.1) and (2.2), we derive:

$$\widehat{\alpha}(\lambda) = \frac{nk}{\sum_{i=1}^{n}\sum_{j=1}^{k}\log(1+\lambda x_{ij})}.$$

$$\widehat{\beta}(\lambda) = \frac{n}{\sum_{i=1}^{n}\log(1+\lambda y_i)}.$$

The MLE of λ, say $\widehat{\lambda}$, is the solution of the following nonlinear equation

$$\frac{n(k+1)}{\lambda} - (\alpha+1)\sum_{i=1}^{n}\sum_{j=1}^{k}\frac{x_{ij}}{1+\lambda x_{ij}} - (\beta+1)\sum_{i=1}^{n}\frac{y_i}{1+\lambda y_i} = 0. \tag{2.3}$$

This equation (2.3) is solved numerically using iterative process as Newton Raphon to get $\widehat{\lambda}$. Then we can get the MLE of $R_{s,k}$ as follows

$$\widehat{R}^{MLE} = \sum_{p=s}^{k}\sum_{q=0}^{k-p}\binom{k}{p}\binom{k-p}{q}(-1)^q\frac{\widehat{\beta}}{\widehat{\beta}+\widehat{\alpha}(p+q)}. \tag{2.4}$$

3. Asymptotic confidence interval

In this section, by obtaining the asymptotic distribution of $R_{s,k}$, the asymptotic confidence interval for $\widehat{R}^{MLE}_{s,k}$, is derived. Because $R_{s,k}$ is a function of the unknown parameters, so by using the asymptotic distribution, the asymptotic variances of the MLE are obtained by the inverse of the observe Fisher information matrix $\mathbf{I} = [I_{ij}] = [\frac{-\partial \ell}{\partial\theta_i\partial\theta_j}]$, where $i,j = 1,2,3$ and $\theta = (\widehat{\alpha},\widehat{\beta},\widehat{\lambda})$. The elements of the observed Fisher information matrix are second partial derivative of log-likelihood function, which can be evaluated as follow:

$$I_{11} = \frac{nk}{\alpha^2},\ I_{12} = 0,\ I_{22} = \frac{n}{\beta^2},\ I_{13} = \sum_{i=1}^{n}\sum_{j=1}^{k}\frac{x_{ij}}{1+\lambda x_{ij}},\ I_{23} = \sum_{i=1}^{n}\frac{y_i}{1+\lambda y_i},$$

$$I_{33} = \frac{n(k+1)}{\lambda^2} + (\alpha+1)\sum_{i=1}^{n}\sum_{j=1}^{k}\left(\frac{x_{ij}}{1+\lambda x_{ij}}\right)^2 + (\beta+1)\sum_{i=1}^{n}\left(\frac{y_i}{1+\lambda y_i}\right)^2.$$

Theorem 3.1. *Suppose that $n,k\to\infty$ and $n/k = p$ then*

$$[\widehat{\alpha}-\alpha\ \widehat{\beta}-\beta\ \widehat{\lambda}-\lambda]^T \xrightarrow{D} N_3(0,\mathbf{I^{-1}}(\alpha,\beta,\lambda)),$$

where $\mathbf{I}(\alpha,\beta,\lambda)$ and $\mathbf{I^{-1}}(\alpha,\beta,\lambda)$ are symmetric matrices as

$$\mathbf{I}(\alpha,\lambda,\mu) = \begin{pmatrix} I_{11} & 0 & I_{13} \\ & I_{22} & I_{23} \\ & & I_{33}\end{pmatrix},\ \mathbf{I^{-1}}(\alpha,\beta,\lambda) = \frac{1}{|\mathbf{I}(\alpha,\beta,\lambda)|}\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ & b_{22} & b_{23} \\ & & b_{33}\end{pmatrix},$$

in which $|\mathbf{I}(\alpha,\beta,\lambda)| = I_{11}I_{22}I_{33} - I_{11}I_{23}^2 - I_{13}^2 I_{22}$,

$$b_{11} = I_{22}I_{33} - I_{23}^2,\quad b_{12} = I_{13}I_{23},\quad b_{13} = -I_{13}I_{22},$$
$$b_{22} = I_{11}I_{33} - I_{13}^2,\quad b_{23} = -I_{11}I_{23},\quad b_{33} = I_{11}I_{22}.$$

Proof. The theorem is proved from the asymptotic normality of the MLE estimates. □

Theorem 3.2. *Suppose that $\widehat{R}^{MLE}_{s,k}$ is the MLE of $R_{s,k}$. So,*

$$(\widehat{R}^{MLE}_{s,k} - R_{s,k}) \xrightarrow{D} N(0, B),$$

where

$$B = \frac{1}{|\mathbf{I}(\alpha, \lambda, \mu)|}\left[(\frac{\partial R_{s,k}}{\partial \alpha})^2 b_{11} + (\frac{\partial R_{s,k}}{\partial \beta})^2 b_{22} + 2(\frac{\partial R_{s,k}}{\partial \alpha})(\frac{\partial R_{s,k}}{\partial \beta}) b_{12}\right], \quad (3.1)$$

$$\frac{\partial R_{s,k}}{\partial \alpha} = \sum_{p=s}^{n}\sum_{q=1}^{k}\binom{k}{p}\binom{k-p}{q}\frac{(-1)^{q+1}\beta(p+q)}{(\alpha(p+q)+\beta)^2}, \quad (3.2)$$

$$\frac{\partial R_{s,k}}{\partial \beta} = \sum_{p=s}^{n}\sum_{q=1}^{k}\binom{k}{p}\binom{k-p}{q}\frac{(-1)^{q}\alpha(p+q)}{(\alpha(p+q)+\beta)^2}. \quad (3.3)$$

Proof. Using Theorem 3.1 and applying delta method, we attain the asymptotic distribution of $\widehat{R}$ as

$$(\widehat{R}^{MLE}_{s,k} - R_{s,k}) \xrightarrow{D} N(0, B),$$

where $B = \mathbf{b^T I^{-1}}(\alpha, \beta, \lambda)\mathbf{b}$ and $b = [\frac{\partial R_{s,k}}{\partial \alpha}\ \frac{\partial R_{s,k}}{\partial \beta}\ \frac{\partial R_{s,k}}{\partial \lambda}]^T = [\frac{\partial R}{\partial \alpha}\ \frac{\partial R}{\partial \lambda}\ 0]^T$, as the rest is defined in 3.1, the Theorem is proved. □

By Theorem 3.2, we construct a $100(1-\gamma)\%$ asymptotic confidence interval of R as:

$$(\widehat{R}^{MLE} - z_{1-\frac{\gamma}{2}}\sqrt{\widehat{B}}, \widehat{R}^{MLE} + z_{1-\frac{\gamma}{2}}\sqrt{\widehat{B}}), \quad (3.4)$$

where z_γ is 100γ-th percentile of $N(0,1)$.

4. Bayes estimation

In this section, we provide the Bayes inference of $R_{s,k}$ where α, β and λ are gamma random variables. So we consider the following priors for α, β and λ,

$$\begin{aligned}
\pi_1(\alpha) &\propto \alpha^{a_1-1}e^{-b_1\alpha}, && \alpha > 0,\ \ a_1, b_1 > 0,\\
\pi_2(\beta) &\propto \beta^{a_2-1}e^{-b_2\beta}, && \beta > 0,\ \ a_2, b_2 > 0,\\
\pi_3(\lambda) &\propto \lambda^{a_3-1}e^{-b_3\lambda}, && \lambda > 0,\ \ a_3, b_3 > 0.
\end{aligned}$$

The joint posterior PDF based on the observed sample, is defined as :

$$\pi(\alpha, \beta, \lambda|\text{data}) = \frac{L(\text{data}|\alpha, \beta, \lambda)\pi_1(\alpha)\pi_2(\underline{)}\pi_2(\beta)\pi_3(\lambda)}{\int_0^\infty\int_0^\infty\int_0^\infty L(\text{data}|\alpha\beta, \lambda)\pi_1(\alpha)\pi_2(\beta)\pi_3(\lambda)d\alpha\ d\beta\ d\lambda}. \quad (4.1)$$

It is impossible to obtain (4.1), analytically. Therefore instead, we propose to approximate it by using two following methods:

- Lindley's approximation,
- MCMC method.

4.1. **Lindley's approximation.** One of the most numerical methods to evaluate the Bayes estimate is Lindley's method, see [5]. This approximat procedure can compute the ratio of two integrals. If $U(\theta)$ is a function of unknown parameters, then under the squared error loss function, the Bayes estimate of $U(\theta)$ can be derived from the following integral representation:

$$\mathbb{E}\big(u(\theta)|\text{data}\big) = \frac{\int u(\theta)e^{Q(\theta)}d\theta}{\int e^{Q(\theta)}d\theta},$$

where $Q(\theta) = \ell(\theta) + \rho(\theta)$, $\ell(\theta)$ is log-likelihood function and $\rho(\theta)$ is the logarithm of prior density of θ. The Lindley's approximation of $\mathbb{E}(u(\theta)|\text{data})$ is given by

$$\mathbb{E}\big(u(\theta)|\text{data}\big) = u + \frac{1}{2}\sum_i\sum_j(u_{ij} + 2u_i\rho_j)\sigma_{ij} + \frac{1}{2}\sum_i\sum_j\sum_k\sum_p \ell_{ijk}\sigma_{ij}\sigma_{kp}u_p\Big|_{\theta=\widehat{\theta}},$$

where $\theta = (\theta_1, \dots, \theta_m)$, $i, j, k, p = 1, \dots, m$, $\widehat{\theta}$ is the MLE of θ, $u = u(\theta)$, $u_i = \partial u/\partial\theta_i$, $u_{ij} = \partial^2 u/\partial\theta_i\partial\theta_j$, $\ell_{ijk} = \partial^3\ell/\partial\theta_i\partial\theta_j\partial\theta_k$, $\rho_j = \partial\rho/\partial\theta_j$, and $\sigma_{ij} = (i,j)$-th element in the inverse of matrix $[-\ell_{ij}]$ all evaluated at the MLE of parameters. For the three parameters case $\theta = (\theta_1, \theta_2, \theta_3)$, Lindley's approximate result in

$$\begin{aligned}\mathbb{E}\big(u(\theta)|\text{data}\big) = u &+ (u_1d_1 + u_2d_2 + u_3d_3 + d_4 + d_5) + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13})\\ &+ B(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23}) + C(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})],\end{aligned}$$

evaluated at $\widehat{\theta} = (\widehat{\theta}_1, \widehat{\theta}_2, \widehat{\theta}_3)$, where

$$\begin{aligned}
d_i &= \rho_1\sigma_{i1} + \rho_2\sigma_{i2} + \rho_3\sigma_{i3},\ i = 1,2,3, \quad d_4 = u_{12}\sigma_{12} + u_{13}\sigma_{13} + u_{23}\sigma_{23},\\
d_5 &= \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22} + u_{33}\sigma_{33}),\\
A &= \ell_{111}\sigma_{11} + 2\ell_{121}\sigma_{12} + 2\ell_{131}\sigma_{13} + 2\ell_{231}\sigma_{23} + \ell_{221}\sigma_{22} + \ell_{331}\sigma_{33},\\
B &= \ell_{112}\sigma_{11} + 2\ell_{122}\sigma_{12} + 2\ell_{132}\sigma_{13} + 2\ell_{232}\sigma_{23} + \ell_{222}\sigma_{22} + \ell_{332}\sigma_{33},\\
C &= \ell_{113}\sigma_{11} + 2\ell_{123}\sigma_{12} + 2\ell_{133}\sigma_{13} + 2\ell_{233}\sigma_{23} + \ell_{223}\sigma_{22} + \ell_{333}\sigma_{33}.
\end{aligned}$$

Now, when $(\theta_1, \theta_2, \theta_3) \equiv (\alpha, \beta, \lambda)$ and $u \equiv u(\alpha, \beta, \lambda) = R_{s,k}$, we have

$$\rho_1 = \frac{a_1 - 1}{\alpha} - b_1,\ \rho_2 = \frac{a_2 - 1}{\beta} - b_2,\ \rho_3 = \frac{a_3 - 1}{\lambda} - b_3 \quad \ell_{11} = -\frac{nk}{\alpha^2},$$

$$\ell_{22} = -\frac{n}{\beta^2},\ \ell_{12} = 0,\ \ell_{13} = \frac{\partial^2\ell}{\partial\alpha\partial\lambda} = \sum_{i=1}^n\sum_{j=1}^k \frac{x_{ij}}{1 + \lambda x_{ij}},\ \ell_{23} = \frac{\partial^2\ell}{\partial\lambda\partial\mu} = \sum_{i=1}^n \frac{y_i}{1 + \lambda y_i},$$

$$\ell_{33} = \frac{\partial^2\ell}{\partial\lambda^2} = \frac{n(k+1)}{\lambda^2} - (\alpha + 1)\sum_{i=1}^n\sum_{j=1}^k\left(\frac{x_{ij}}{1 + \lambda x_{ij}}\right)^2 - (\beta + 1)\sum_{i=1}^n\left(\frac{y_i}{1 + \lambda y_i}\right)^2.$$

σ_{ij}, $i, j = 1,2,3$ are obtained by using ℓ_{ij}, $i, j = 1,2,3$ and

$$\ell_{111} = \frac{2nk}{\alpha^3},\ \ell_{222} = \frac{2n}{\beta^3},\ \ell_{133} = \sum_{i=1}^n\sum_{j=1}^k\left(\frac{x_{ij}}{1 + \lambda x_{ij}}\right)^2,\ \ell_{233} = \sum_{i=1}^n\left(\frac{y_i}{1 + \lambda y_i}\right)^2,$$

$$\ell_{333} = \frac{2n(k+1)}{\lambda^3} - (\alpha + 1)\sum_{i=1}^n\sum_{j=1}^k\left(\frac{x_{ij}}{1 + \lambda x_{ij}}\right)^3 - (\beta + 1)\sum_{i=1}^n\left(\frac{y_i}{1 + \lambda y_i}\right)^3,$$

and the other $\ell_{ijk} = 0$. Furthermore, $u_3 = u_{i3} = 0$, $i = 1, 2, 3$, and u_1, u_2 are explained in (3.2) and (3.3), respectively. Also,

$$u_{11} = \sum_{p=s}^{n}\sum_{q=1}^{k}\binom{k}{p}\binom{k-p}{q}\frac{(-1)^{q+1}\beta(p+q)^2}{(\alpha(p+q)+\beta)^3},$$

$$u_{12} = u_{21} = \sum_{p=s}^{n}\sum_{q=1}^{k}\binom{k}{p}\binom{k-p}{q}\frac{(-1)^{q}(p+q)(\beta-\alpha(p+q))}{(\alpha(p+q)+\beta)^3},$$

$$u_{22} = \sum_{p=s}^{n}\sum_{q=1}^{k}\binom{k}{p}\binom{k-p}{q}\frac{(-1)^{q+1}2\alpha(p+q)}{(\alpha(p+q)+\beta)^3}.$$

Therefore,

$$d_4 = u_{12}\sigma_{12}, \qquad d_5 = \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22}),$$

$$A = \ell_{111}\sigma_{11} + \ell_{331}\sigma_{33},\ B = \ell_{222}\sigma_{22} + \ell_{332}\sigma_{33},\ C = 2\ell_{133}\sigma_{13} + 2\ell_{233}\sigma_{23} + \ell_{333}\sigma_{33}.$$

So, the Bayes estimate of $R_{s,k}$ resulted by

$$\begin{aligned}\widehat{R}_{s,k}^{Lin} = R + [u_1d_1 + u_2d_2 + d_4 + d_5] + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12})\\ + B(u_1\sigma_{21} + u_2\sigma_{22}) + C(u_1\sigma_{31} + u_2\sigma_{32})].\end{aligned} \tag{4.2}$$

Notice that all parameters should be estimated at $(\widehat{\alpha}, \widehat{\beta}, \widehat{\lambda})$.

Because the Bayesian credible interval, applying the Lindley's approximation, is not available, we force to use MCMC method. Utilizing this method, Bayes estimate is approximated and associated HPD credible interval is constructed.

4.2. **MCMC method.** From (4.1), the posterior pdf of α, β and λ are as follows:

$$\alpha|\lambda, \text{data} \sim \Gamma\big(n + a_1, b_1 + \sum_{i=1}^{n}\sum_{j=1}^{k}\log(1+\lambda x_{ij})\big),$$

$$\beta|\lambda, \text{data} \sim \Gamma\big(nk + a_2, b_2 + \sum_{i=1}^{n}\log(1+\lambda y_i)\big),$$

$$\pi(\lambda|\alpha,\beta,\text{data}) \propto \lambda^{n(k+1)+a_3-1}e^{-b_3\lambda}\left(\prod_{i=1}^{n}\prod_{j=1}^{k}(1+\lambda x_{ij})^{-\alpha}\right)\left(\prod_{i=1}^{n}(1+\lambda y_i)^{-\beta}\right).$$

Because we cannot reduce the posterior pdf of λ analytically to a well known distribution, so we force to use the Metropolis-Hastings method to generate random samples form it. Therefore, we propose the Gibbs sampling algorithm as follows:

1. Start with the initial value $(\alpha_{(0)}, \beta_{(0)}, \lambda_{(0)})$.
2. Set $t = 1$.
3. Generate $\lambda_{(t)}$ from $\pi(\lambda|\alpha_{(t-1)}, \beta_{(t-1)}, \text{data})$, using Metropolis-Hastings method.
4. Generate $\alpha_{(t)}$ from $\Gamma\big(n + a_1, b_1 \sum_{i=1}^{n}\sum_{j=1}^{k}\log(1+\lambda_{(t-1)}x_{ij})\big)$.
5. Generate $\beta_{(t)}$ from $\Gamma\big(nk + a_2, b_2 + \sum_{i=1}^{n}\log(1+\lambda_{(t-1)}y_i)\big)$.
6. Compute $R_{(t)s,k} = \sum_{p=s}^{k}\sum_{q=0}^{k-p}\binom{k}{p}\binom{k-p}{q}\frac{(-1)^q\beta_{(t)}}{\alpha_{(t)}(p+q)+\beta_{(t)}}$.

(7) Set $t = t + 1$.
(8) Repeat steps 3-7, T times.

This above algorithm is used to evaluate the Bayes estimate of $R_{s,k}$ under the squared error loss function. Therefore, the MCMC Bayes estimate can be resulted by

$$\widehat{R}_{s,k}^{MC} = \frac{1}{T}\sum_{t=1}^{T} R_t. \tag{4.3}$$

In addition, applying the method of Chen and Shao 1999, we construct a $100(1-\gamma)\%$ HPD credible interval of R using the idea of [2].

5. Simulation Study

We consider the performance of different estimates by using the Monte Carlo simulations. The different estimates, in terms of mean squared errors (MSEs) are compared together and the different confidence intervals, in terms of average confidence lengths are compared together. Based on 1000 replications, all results are gathered. The parameter values $(\theta, \lambda, \alpha) = (1, 1, 1)$ are used to obtain the simulation results. We derive MLE of $R_{s,k}$ by (2.4) and asymptotic confidence interval for it using (3.4). Also, the Bayesian inference is considered by assuming two priors as Prior 1: $a_j = 0,\ b_j = 0,\ j = 1, 2$, Prior 2: $a_j = 1,\ b_j = 1,\ j = 1, 2$. Under the above hypotheses, the MSEs of Bayesian estimates of $R_{s,k}$, via Linldey's approximation and MCMC method are derived by (4.2) and (4.3), respectively. Also, we derived the 95% HPD intervals for $R_{s,k}$. The simulation results are given in Table 1.

From Table 1, we observed that the best performance, in terms of MSE, belong to informative priors. Furthermore, the performance of Bayes estimates which obtained by MCMC method are generally better than those obtained by Lindleys approximation. Also, we observed that the best performance among the different intervals belong to HPD intervals based on informative priors.

Table 1. Simulation results

n	$R_{s,k}$			Prior 1			Prior 2		
		MLE		Lindley	MCMC		Lindley	MCMC	
		MSE	Length	MSE	MSE	HPD	MSE	MSE	HPD
10	(3,5)	0.0254	0.3912	0.0229	0.0039	0.3512	0.0237	0.0026	0.3365
	(2,4)	0.0317	0.4125	0.0276	0.0094	0.4098	0.0303	0.0064	0.3876
20	(3,5)	0.0178	0.3542	0.0166	0.0019	0.3355	0.0177	0.0015	0.3021
	(2,4)	0.0248	0.4098	0.0245	0.0044	0.3987	0.0224	0.0036	0.3711
30	(3,5)	0.0157	0.3365	0.0156	0.0013	0.3287	0.0148	0.0011	0.2912
	(2,4)	0.0234	0.3777	0.0231	0.0030	0.3542	0.0216	0.0027	0.3444
40	(3,5)	0.0153	0.3023	0.0152	0.0009	0.2889	0.0146	0.0008	0.2768
	(2,4)	0.0225	0.3542	0.0221	0.0025	0.3324	0.0211	0.0022	0.3122
50	(3,5)	0.0153	0.2877	0.0152	0.0008	0.2531	0.0147	0.0007	0.2436
	(2,4)	0.0222	0.3333	0.0219	0.0019	0.3165	0.0211	0.0018	0.3054

References

1. Bhattacharyya G. K. and Johnson R. A. (1974), Estimation of reliability in multicomponent stress-strength model. *Journal of the American Statistical Association,* **69**, 966-970.
2. Chen M. H. and Shao Q. M. (1999), Monte Carlo estimation of Bayesian Credible and HPD intervals. *Journal of Computational and Graphical Statistics,* **8**, 69-92.
3. Kohansal, A. (2019), On estimation of reliability in a multicomponent stress-strength model for a Kumaraswamy distribution based on progressively censored sample, *Statistical Papers,* **60**, 2185-2224.
4. Kohansal A. and Shoaee S. (2019), Bayesian and classical estimation of reliability in a multi-component stress-strength model under adaptive hybrid progressive censored data, *Statistical Papers,* Accepted. DOI: 10.1007/s00362-019-01094-y.
5. Lindley D. V. (1980), Approximate Bayesian methods. *Trabajos de Estadistica,* **3**, 281-288.